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The characteristics of runoff formation on forest and field sections of drainage basins are examined. A mathematical model is proposed which takes into account the main thermophysical processes in the zone of aeration and can be used for continuous calculation of snowmelt, rainfall, and snowmelt-rainfall runoff on rivers with various degrees of forest coverage. The parameters of the model are determined by optimization methods. A check of the model obtained on a number of small drainage basins of the forest zone showed a good convergence of the actual and calculated hydrographs. The relationship of the runoff volumes from field and forest sections agrees with physical concepts.

A number of runoff formation models are presently used in hydrological forecasts. We can first of all include among them the model of formation of rainfall floods [5] and model of formation of the spring flood hydrograph [2]. Experience shows that when realizing these models great difficulties arise in the transition periods, when the water-absorbing properties of the drainage basin change very markedly as a result of freezing and thawing of the soil. In connection with this it is not possible to construct a continuous scheme of short-range discharge forecasts.

In this article a model is proposed which takes into account the main thermophysical processes occurring in the zone of aeration, thanks to which it can be used for continuous calculation of snowmelt, rainfall, and snowmelt-rainfall runoff. This model is a generalization of the model obtained earlier for a completely forested drainage basin [1] to the case of partially forested river basins.

The conditions of runoff formation on forest and field sections differ substantially. One of the main factors causing great differences in the water-absorbing properties in the field and forest is the presence of a small (of the order of 5-10 cm) very loose surface layer of soil in the forest. The data given in [7] on the texture of various soils shows that from a depth of 5-10 cm downward the porosity of soil on field and forest sections differs very insignificantly, whereas in the 0-10-cm layer these differences are very substantial. This circumstance permits simplifying somewhat the model of runoff formation for partially forested drainage basins.

We will tentatively transfer the line of the ground surface in the forest to the interface between the upper (5-10 cm) very permeable layer and the underlying soil stratum with considerably smaller porosity. In other words, we will assume that in the forest all water arriving on the soil surface is instantaneously absorbed by the upper 5-10-cm layer. A part of this water seeps into the underlying layer and a part flows into the rivulet network, forming a runoff close in time to the surface runoff. In that case the model can be constructed so that the relations for calculating the majority of the components of the balance in the field and forest will differ only by the coefficients.

We will isolate the upper soil layer with thickness  $z$  and maximum moisture capacity  $\omega_m$ . For forest sections the upper boundary of this layer will be the line of the tentative ground surface (Fig. 1). If we assume that phase transitions occur only at the freezing or thawing front and all moisture freezes in the zone of negative temperatures, we can comparatively simply obtain the balance relationships of moisture for the solid and liquid phases

$$\frac{dW_{li}}{dt} = v_i + r_{li} \quad (i = 1, 2, \dots, 5), \quad (1)$$

$$\frac{dW_{t,i}}{dt} = r_{t,i} \quad (i = 2, 4), \quad (2)$$

where  $W_{t,i}$ ,  $W_{f,i}$  are the storage of liquid and solid components of moisture in the  $i$ -th layer,  $v_i$  is the rate of change of the liquid component as a consequence of water exchange with the upper and lower layers,  $r_{t,i}$ ,  $r_{f,i}$  is the rate of change of the thawed and frozen components as a result of phase transitions at the upper or lower boundaries of the layer.

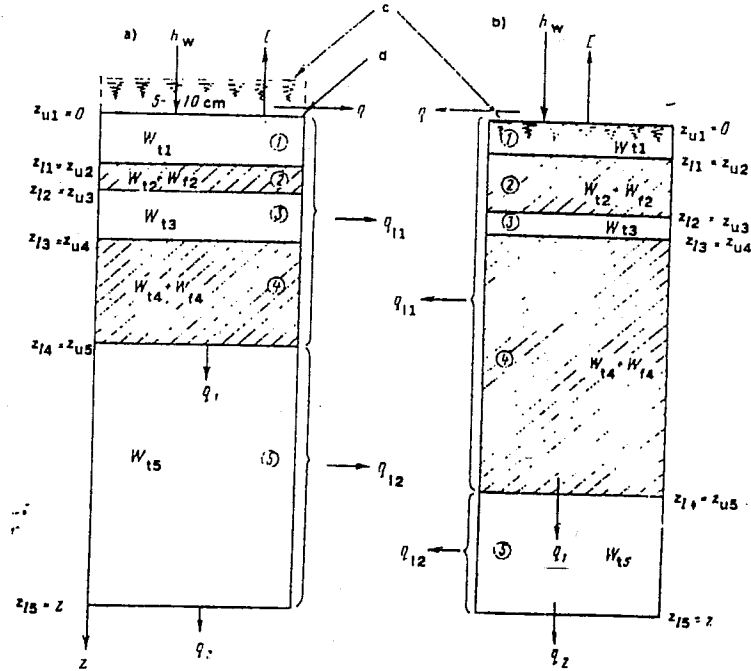


Fig. 1. Schematic section of soil for forest (a) and field (b) sections of drainage basin: c) line of ground surface; d) line of tentative ground surface; 1, 3, 5) layers of thawed soil; 2, 4) frozen layers.

Following Fig. 1, we will write the expressions for calculating the components of the balance (thawing of the soil from below is not taken into account):

$$r_{t,i} = \left| \ln \left( \frac{i+1}{2} \right) - \ln \left( \frac{i}{2} \right) \right| \frac{dZ_{l,i}}{dt} \frac{W_{t,i+1}}{\Delta Z_{l,i+1}} - \frac{dZ_{u,i}}{dt} \frac{W_{t,i}}{\Delta Z_i} \quad (i = 1, 2, \dots, 5), \quad (3)$$

$$r_{f,i} = \frac{dZ_{u,i}}{dt} \frac{W_{t(i+1)}}{\Delta Z_{i+1}} - \frac{dZ_{u,i}}{dt} \frac{W_{f,i}}{\Delta Z_i} \quad (i = 2, 4), \quad (4)$$

$$v_i = \frac{\Delta Z_i}{Z_{l,i}} (h_w - E - q - q_{l,i} - q_1) \quad (i = 1, 2, \dots, 4), \quad (5)$$

$$v_5 = q_1 - q_2 - q_{l,5}, \quad (6)$$

where  $Z_{l,i}$ ,  $Z_{u,i}$  are respectively the lower and upper boundaries of the  $i$ -th layer,  $W$  is the total moisture storage,

$$\Delta Z_i = Z_{l,i} - Z_{u,i}$$

$h_w$  is the water supply to the soil surface,  $E$  is the evapotranspiration from the drainage basin,  $q$  is the surface runoff,  $q_{I1}$ ,  $q_{I2}$  are the interflow from the frozen and lower nonfrozen zones,  $q_1$ ,  $q_2$  are the outflow of moisture through the lower boundaries of the frozen and nonfrozen zones,  $\text{int}(x)$  is the integral part of variable  $x$ .

The interfaces between the frozen and thawed layers of soil were calculated by an approximate relation [6] obtained from equations of heat transport under the following main assumptions: inflow of heat (cold) from below is absent; the distribution of temperatures in the frozen (thawed) layer and in the snow cover is linear; at a negative (positive) temperature all moisture is in a solid (liquid) state; the water percolating during thawing does not participate in heat-transport processes:

$$Z(t + dt) = -\lambda H/\lambda_s + \{[\lambda H/\lambda_s + Z(t)]^2 + 2\lambda |T| dt \rho_w L_w\}^{1/2}, \quad (7)$$

where  $H$  and  $T$  are respectively the height of the snow cover and air temperature during the investigated time interval  $dt$ ;  $Z(t)$  and  $Z(t + dt)$  are the positions of the interface by the start and end of the calculated time interval, respectively;  $w$  is the moisture volume percentage at the freezing (thawing) front;  $\rho_w$  is the density of water;  $L$  is the latent heat of fusion of ice;  $\lambda_s$  is the coefficient of thermal conductivity of frozen (during freezing) and thawed (during thawing) soil.

Relationships (1)-(7) are valid both for the field and forest sections of the drainage basin. We will assume that the structure of the relationships for calculating the water supply, evaporation, interflow, and outflow of moisture to underlying layers is the same for the field and forest sections. Then for their calculation we can use the relationships obtained earlier for completely forested drainage basins [1]. However, the parameters of these relationships can be different for the field and forest sections.

Another fundamentally important distinctive feature of the formation of the snowmelt runoff in the field is the formation of "barrier" layers, as a result of which infiltration on such sections is almost completely absent. Such layers, as a rule, are not formed in the forest. In connection with this, in the period of melting of the snow the relationships for calculating the dynamics of the surface runoff in the field and forest should differ fundamentally.

As was noted above, by surface runoff in the forest we will mean the runoff in the uppermost 5-10-cm soil layer. The infiltration rate in this case will be determined by the water-absorbing properties of the lower-lying soil layer. If we take into account also the detention of water in the upper layer, we can obtain a relationship for calculating the surface runoff in the forest

$$q_{to} = \begin{cases} (h_w - I) \left\{ 1 - \exp \left[ -m \sum (h_w - I - E) \right] \right\}, & I < h_w \\ 0, & I \geq h_w \end{cases} \quad (8)$$

where  $I$  is the infiltration rate through the tentative ground surface,  $m$  is the detention parameter.

For a field we will assume that infiltration occurs only on sections where a "barrier" layer has not formed. On the remaining part infiltration is absent and losses are formed due to surface detention, i.e.,

$$q_n = \begin{cases} (h_w - I) s_k F_k + h_w s (1 - F_k), & I < h_w \\ h_w s (1 - F_k), & I \geq h_w \end{cases}$$

$$s_k = 1 - \exp \left\{ -m \sum (h_w - I - E) \right\}, \quad (9)$$

$$s = 1 - \exp \left\{ -m \sum (h_w - E) \right\},$$

where  $F_k$  is the portion of the area of field sections on which a "barrier" layer was not

formed.

The infiltration rate for the field and forest sections was calculated by the relationship

$$I = k(\omega_{mp} - \omega)/Z + i(\omega_t/\omega_{mp})^n(1 + 8\omega_t)^{-2}, \quad (10)$$

where  $\omega_{mp}$  is the maximum possible productive moisture content,  $\omega_t$  is the volume content of liquid moisture,  $\omega_f$  is the volume ice content,  $k, i, n$  are parameters.

$$\omega = \omega_t + \omega_f,$$

A substantiation of expression (10) is given in [1].

Certain critical relationships of the water-absorbing characteristics of the soil are needed for the formation of the "barrier" layer. According to V. D. Komarov's laboratory investigations [4], a "barrier" layer is formed only when a certain critical soil temperature is reached, the value of which depends on the soil moisture content. In practice the depth of freezing is often used as an indirect characteristic of soil temperature. In [3] for example, the author gives the portion of the area on which a "barrier" layer formed as a function of the product of the basin's average depths of freezing and soil moisture content obtained from experimental data.

The soil moisture content ( $\omega$ ) and depth of freezing ( $Z$ ) vary markedly over a drainage basin. Suppose that for field sections of a particular drainage basin there exists a certain critical value  $u_k = (Z\omega)_k$  on exceeding which the soil is practically impermeable. If we assume that  $Z$  and  $\omega$  at each point of the basin are independent and stable distribution curves exist for them, then we can write the following expression for calculating the portion of the area on which a "barrier" layer is not formed:

$$F_k(u \leq u_k) = \int_0^\infty \int_0^{u_k/\omega} f(\omega)f(Z)d\omega dZ, \quad (11)$$

where  $f(\omega)$  and  $f(Z)$  are the distribution curves of the soil moisture content and depth of freezing on field sections of the drainage basin. To describe these curves we will use the rather flexible one-parameter gamma distribution of the modulus coefficients:

$$f(K) = \frac{\alpha^\alpha}{\Gamma(\alpha)} K^{\alpha-1} e^{-\alpha K}, \quad (12)$$

where  $K$  is the modulus coefficient, equal to  $Z_1/\bar{Z}$  or  $\omega_1/\bar{\omega}$ ;  $\alpha$  is a parameter of the distribution, equal to  $1/C_v^2$ ;  $C_v$  is the coefficient of variation of the corresponding variable;  $\bar{Z}, \bar{\omega}$  are the mean values of the variables with respect to the drainage basin.

Having substituted Eq. (12) into (11) and integrated the latter, we obtain

$$F_k = \frac{\alpha_\omega^\alpha}{\Gamma(\alpha_\omega)\Gamma(\alpha_z)} \int_0^\omega K_\omega^{\alpha_\omega-1} e^{-\alpha_\omega K_\omega} \Gamma_g(\alpha_z) dK_\omega, \quad (13)$$

where  $\alpha_\omega$  and  $\alpha_z$  are parameters of the distribution curves of the moisture content and depth of freezing, respectively,  $K_\omega$  is the modulus coefficient of the soil moisture content;  $\Gamma_g(\alpha_z)$  is an incomplete gamma function, where  $g = \alpha_z u_k / \bar{Z} \bar{\omega} K_\omega$ .

Equation (13) is too cumbersome for practical calculations. Without substantial loss of accuracy we can assume that the  $\alpha_z$  are only integers. Taking into account also that the nonuniformity of the distribution of depths of freezing for the forest zone is considerably greater than that of the soil moisture content, we will eliminate the effect of the variability of the moisture content over the basin, i.e., we will set  $\alpha_\omega \rightarrow \infty$ . In this case we

can simplify Eq. (13):

$$F_k = 1 - e^{-\alpha_z K_k} \sum_{i=1}^{\alpha_z} (\alpha_z / K_k)^{\alpha_z - i} \Gamma(\alpha_z - i + 1), \quad (14)$$

where  $K_k = u_k / \bar{z}\bar{\omega}$ .

Numerical experiments showed that the differences of the calculations by Eqs. (13) and (14) when  $\alpha_z > \alpha_z$  (if  $\alpha_z \leq 6$ ) are negligible, and Eq. (14) was used thereafter.

When calculating the change of  $F_k$  in the flood period the soil moisture content in Eq. (14) was kept unchanged from the start of melting of the snow and the depth of freezing was reduced by the amount of thawing of the soil by this time. Thus  $F_k$  increased up to the end of the flood, approaching unity.

Komarov for calculating the maximum possible losses (for a practically unlimited water equivalent of snow) suggested the following relationship:

$$P_m = A e^{-\alpha \bar{z}\bar{\omega}} e^{-\beta \bar{z}\bar{\omega}} = I(\bar{\omega}) e^{-\beta \bar{z}\bar{\omega}}, \quad (15)$$

where  $A$ ,  $\alpha$ ,  $\beta$  are parameters.

Integrating Eq. (9) when  $s_k = s = 0$ ,  $\alpha_z = 1$  and  $F_k = \text{const}$ , we can determine the maximum possible losses during the flood:

$$P_m = \left( \int_0^T I dt \right) (1 - e^{-\alpha \bar{z}\bar{\omega}}). \quad (16)$$

In Eq. (16) the first cofactor represents the potential total infiltration losses and the second the portion of the area on which they are formed. Equations (15) and (16) are similar in form, although each of the cofactors is determined on the basis of different relationships. Consequently, we can assume that Eq. (15) also indirectly takes into account the portion of impermeable sections for the case when the soil moisture content is distributed uniformly over the area and the depth of freezing is close to a gamma distribution with parameter  $\alpha_z = 1$ .

All balance equations both for the field and forest were solved separately for snow-covered sections of the drainage basin and sections freed from snow. The surface runoff and interflow from the field and forest sections were determined as the weighted average values with consideration of snow coverage. In calculating the runoff hydrograph at the basin outlet transformation was accomplished differently depending on the value of the coefficient of forest coverage ( $\gamma$ ). For practically completely forest or field drainage basins ( $0.25 > \gamma > 0.75$ ) the weighted average surface and interflow components of the runoff for the entire basin were transformed. In the remaining cases the forest and field components of the surface runoff were transformed separately, and the interflow was summed without additional transformations.

The model obtained includes a number of coefficients which can vary for different particular drainage basins. Some of these coefficients take into account the effect of various zonal factors of runoff formation and vary little for basins located within the limits of the same zone. Other parameters reflect the effect of local characteristics of runoff formation and can substantially differ for different basins located even within homogeneous zones. In connection with this, the success of using the model for particular drainage basins depends in many respects on the reliability of estimating the coefficients of the model reflecting both zonal conditions of runoff formation and local characteristics of the drainage basin.

Testing and further development of the model were performed on the basis of the data of five small (drainage area 5000-10,000 km<sup>2</sup>) drainage basins of the forest zone in the Volga and Dniepr River basins for which a flood in the presence of a relatively stable snow cover is characteristic. The coefficient of forest coverage for these drainage areas varied from 0.25 to 0.80. Data of standard observations of the snow cover, daily air temperatures and saturation deficit, precipitation, and water discharges at the basin outlet were used. Data on the variation of the depth of freezing and thawing of the soil were used only for a qualitative analysis of the results of calculating these characteristics.

An optimization method with a quality criterion equal to the sum of the squares of the deviations of the actual and calculated water discharges was used for determining the parameters [5]. The parameters were optimized in two stages. At first the most significant parameters which were practically impossible to measure or calculate directly were optimized. Primarily the coefficients of Eqs. (8)-(10) and the transformation parameters belong to this group. The number of these parameters for drainage basins with various forest coverage can vary from 9 to 13. The other parameters of the model (maximum soil moisture capacity, evaporation parameter, thawing coefficients) were not included in optimization, and their values were determined by actual measurements or from the balance relationships. Combined optimization of the parameters of the first and second groups was performed in the second stage.

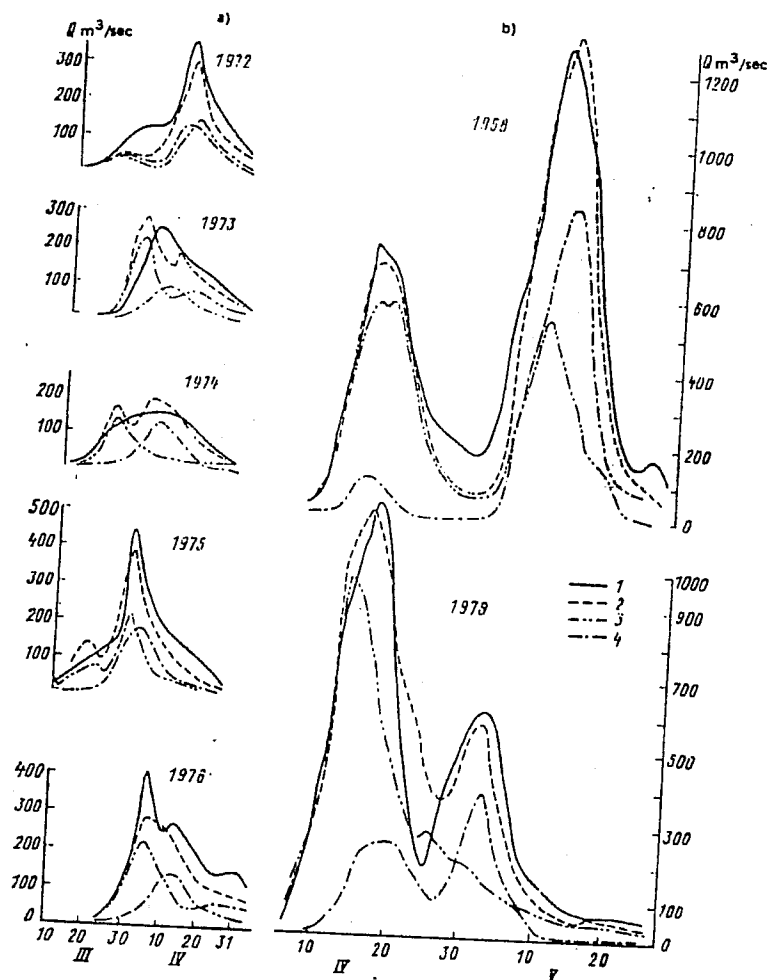


Fig. 2. Actual (1) and calculated (2, total; 3, from field; 4, from forest) runoff hydrographs. a) Dniepr River, Dorogobuzh city; b) Cheptsya River, Glazov city.

This approach made it possible to obtain for all investigated drainage basins sufficiently stable values of the parameters, which is confirmed by the practically identical accuracy of calculating the discharges for dependent and independent (the data were not used in optimization) samples. This is attested to also by the weak variability of the majority of parameters on passing from basin to basin. Therefore, we can expect that when the model is extended to other drainage basins of the forest zone it will be possible to reduce the number of parameters being optimized.

The actual and calculated runoff hydrographs for two drainage basins, shown in Fig. 2, give an idea about the accuracy of calculating discharges during the spring flood. An analogous accuracy was obtained also for rainfall and snowmelt-rainfall floods. Figure 2 shows also the hydrographs formed on field and forest sections of the drainage basins. The coefficients of forest coverage of the drainage basins of the Cheptsya and Dniepr Rivers to the basin outlets are respectively equal to 0.40 and 0.42. Considerable phase and amplitude differences of the hydrographs from the field and forest sections are clearly observed on

the graphs. Thus if we regard a drainage basin as purely field or forest, we can hardly attain a satisfactory accuracy of calculating the hydrographs at the basin outlet. Calculations showed that such an approximation is acceptable for coefficients of forest coverage within  $0.25 < \gamma < 0.80$ .

The depths of runoff during the flood from the field sections of these basins calculated by the proposed model exceed the corresponding values for the forest sections for all years. The average coefficients of runoff from the field and forest sections of the basin of the Cheptsa River were equal to 0.76 and 0.44 and for the Dniepr 0.77 and 0.62. The same such runoff coefficients for the Cheptsa River were obtained in [7] on separating the total runoff during the flood into field and forest components by the method proposed by Komarov. The ratio of these components varies from year to year within wide limits (for the investigated samples, from 1.03 to 1.9).

In conclusion we can note that the model obtained takes into account the characteristics of runoff formation on field and forest sections of a drainage basin and can be used for calculating the snowmelt, rainfall, and snowmelt-rainfall runoff for small rivers of the forest zones with various degrees of forest coverage.

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